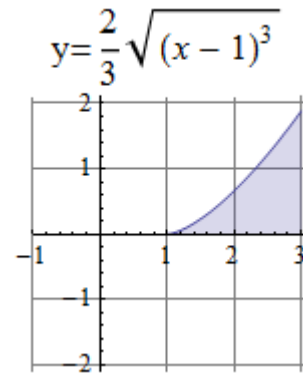


Calculo Integral. Ejercicios de repaso unidad 4.

Aplicaciones geométricas de la integral definida.

Para la región sombreada obtener:

- Área
- Perímetro
- Volumen del sólido que se genera al girar alrededor del eje x
- Centro de masa.



$$\begin{aligned} \text{Área} &= \int_1^3 \frac{2}{3} \sqrt{(x-1)^3} dx = \frac{2}{3} \int_1^3 (x-1)^{\frac{3}{2}} dx = \\ &= \frac{2}{3} \frac{(x-1)^{\frac{5}{2}}}{\frac{5}{2}} \Big|_1^3 = \frac{4}{15} (x-1)^{\frac{5}{2}} \Big|_1^3 = 1.5084 - 0 \\ &= \boxed{1.5084} \end{aligned}$$

$$\begin{aligned} \text{Perímetro} &= L_1 + L_2 + L_3 \\ &= 2 + \frac{4\sqrt{2}}{3} + 2.7974 = \boxed{6.6} \end{aligned}$$

$$\begin{aligned} L_3 \quad y &= \frac{2}{3} (x-1)^{\frac{3}{2}} \\ y' &= \frac{2}{3} \cdot \frac{3}{2} (x-1)^{\frac{1}{2}} = \sqrt{x-1} \end{aligned}$$

$$\begin{aligned} L_3 &= \int_1^3 \sqrt{1+x-1} dx = \int_1^3 \sqrt{x} dx = \int_1^3 x^{\frac{1}{2}} dx \\ &= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \Big|_1^3 = \frac{2}{3} x^{\frac{3}{2}} \Big|_1^3 = 3.4641 - \frac{2}{3} = 2.7974 \end{aligned}$$

$$V = \pi \int_1^3 \left(\frac{2}{3} \sqrt{(x-1)^3} \right)^2 dx = \pi \int_1^3 \frac{4}{9} (x-1)^3 dx$$

$$\frac{4\pi}{9} \int_1^3 (x-1)^3 dx = \frac{\pi}{9} \left[(x-1)^4 \Big|_1^3 \right] = \frac{\pi}{9} [16 - 0]$$

$$V = \frac{16\pi}{9} u^3 \approx \boxed{5.58 u^3}$$

Centro de masa

$$y = \frac{2}{3} \sqrt{(x-1)^3} \text{ en } [1, 3]$$

$$A = \frac{16\sqrt{2}}{15} \approx 1.5$$

$$\bar{x} = \frac{1}{\frac{16\sqrt{2}}{15}} \int_1^3 x \left(\frac{2}{3} \sqrt{(x-1)^3} \right) dx = \frac{15}{16\sqrt{2}} \cdot \frac{2}{3} \int_1^3 x \sqrt{(x-1)^3} dx$$

$$\bar{x} = \frac{5}{8\sqrt{2}} \int_1^3 x \sqrt{(x-1)^3} dx = \frac{5}{8\sqrt{2}} \left[\frac{2}{5} x (x-1)^{2.5} - \frac{4}{35} (x-1)^{3.5} \right]_1^3$$

$$\frac{5}{8\sqrt{2}} [5.4952 - 0] \approx \boxed{2.4}$$

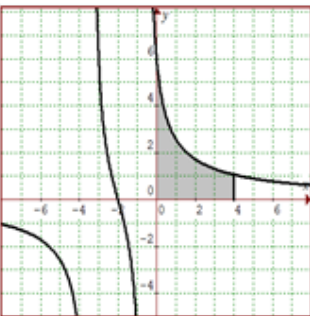
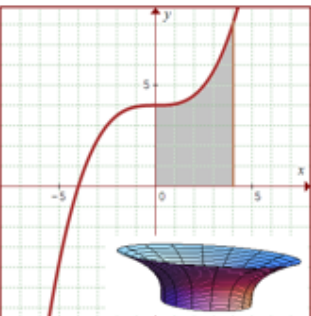
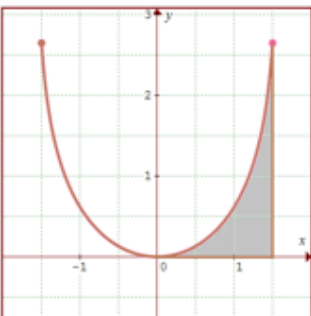
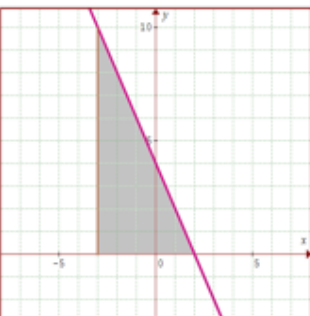
$$\int \frac{2x}{5} (x-1)^{7/2} = \frac{4}{35} (x-1)^{7/2}$$

$$\bar{y} = \frac{1}{\frac{32\sqrt{2}}{15}} \int_1^3 \left(\frac{2}{3} \sqrt{(x-1)^3} \right)^2 dx = \frac{15}{32\sqrt{2}} \int_1^3 \frac{4}{9} (x-1)^3 dx$$

$$\frac{5}{24\sqrt{2}} \int_1^3 (x-1)^3 dx = \frac{5}{96\sqrt{2}} \left[(x-1)^4 \right]_1^3$$

$$= \frac{5}{96\sqrt{2}} [16 - 0] = 0.58 \quad (2.4, 0.58)$$

Ejercicios de Repaso

<p>Área</p> $y = \frac{6x + 12}{x^2 + 4x + 2}$ 	<p>Volumen al girar en eje x $y = \frac{x^3}{16} + 4$</p> 	<p>Perímetro $y = -\ln(\cos x)$</p> 	<p>Centro de masa $y = 4 - 2x$</p> 
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