CÁLCULO INTEGRAL. UNIDAD 3. EJERCICIOS DE REPASO SUSTITUCIÓN Y POTENCIAS TRIGONOMETRICAS.
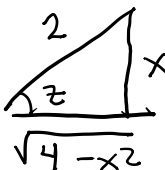
Potencias trigonométricas

$$\int Sen^5 2x\, dx \quad = \frac{-1}{2}\int Sen^4 2x \cdot \boxed{-Sen2x\cdot 2}\, dx$$

$$= -\frac{1}{2}\int (1-\cos^2 2x)^2 \; (-sen\,2x\cdot 2)\, dx$$

$$sen^2 v = 1 - \cos^2 v$$
$$Sen^4 v = (1-\cos^2 v)^2$$

$$= \int \left(-\frac{1}{2} + \cos^2 2x - \frac{1}{2}\cos^4 2x\right)(-sen\,2x\cdot 2)\, dx$$

$$= \int\left[\frac{1}{2}(sen\,2x\cdot 2) + \cos^2 2x(-sen\,2x\cdot 2) - \frac{1}{2}\cos^4 2x(-sen\,2x\cdot 2)\right]dx$$

$$= -\frac{1}{2}\cos 2x + \frac{\cos^3 2x}{3} - \frac{1}{2}\frac{\cos^5 2x}{5} + C$$

$$= -\frac{1}{2}\cos 2x + \frac{\cos^3 2x}{3} - \frac{\cos^5 2x}{10} + C$$

Sustitución trigonométrica

$$\int \sqrt{4-x^2}\, dx = \int 2\cos z\, dz \cdot 2\cos z\cdot dz = 4\int \cos^2 z\, dz$$

$$= 4\int \frac{1+\cos 2z}{2}\, dz = 2\int (1+\cos 2z)\, dz$$

$$= 2z + sen\,2z + C = 2z + 2\,sen\,z\cdot \cos z + C$$

$$= 2\,Arcsen\frac{x}{2} + 2\frac{x}{2}\cdot\frac{\sqrt{4-x^2}}{2}$$

$$= 2\,Arcsen\frac{x}{2} + \frac{1}{2}x\sqrt{4-x^2} + C$$

$$sen(z) = \frac{x}{2}$$
$$\cos z = \frac{\sqrt{4-x^2}}{2}$$
$$x = 2\,sen\,z$$
$$dx = 2\cos z\cdot dz$$

Sustitución por Cambio de variable

$$\int (\sqrt{x}+1)^8 dx = \int u^8\cdot (2u-2)\, du = \int (2u^9 - 2u^8)\, du$$

$$\sqrt{x}+1 = u$$
$$\sqrt{x} = u-1$$
$$x = (u-1)^2$$
$$x = u^2 - 2u + 1$$
$$dx = (2u-2)\, du$$

$$= \frac{2u^{10}}{10} - \frac{2u^9}{9} + C$$

$$= \frac{1}{5}(\sqrt{x}+1)^{10} - \frac{2}{9}(\sqrt{x}+1)^9 + C$$

EJERCICIOS DE REPASO

Potencias trigonométricas

$$\int Cos^3 7x \, dx$$

Sustitución trigonométrica

$$\int \frac{dx}{\sqrt{4 - 4x^2}}$$

Cambio de variable

$$\int (x + 1)^8 x^2 \, dx$$